

Help prevent infections and disruptions from your studies

- ▶ Wear masks inside if you can
- ▶ Do not come onto campus if you have COVID-19 symptoms

Last year's videos are available on Blackboard.

Housekeeping: Office Hours

Office hours mean I'm in my office for people to drop in and chat. Take advantage of them! I tried to pick office hours with few/no clashes with level 3 lectures, and set at half-hours to make it likely you can make part of them.

Office Hours:

- ▶ Wednesday 11:30-12:30 (online Blackboard Collaborate)
- ▶ Friday 14:30-15:30 (in my office, J06b)

These are subject to change; if you'd like to come a few times and those will never work, let me know, and maybe I'll change them! You can also email to try to set up another time to drop in.

Section 1.3: Inequalities

Covered 1.1 Yesterday. 1.2 is worked examples!

Lemma (Triangle Inequality)

For $z, w \in \mathbb{C}$ we have

$$||z| - |w|| \leq |z + w| \leq |z| + |w|$$

$$||z| - |w|| \leq |z - w| \leq |z| + |w|$$

Proof.

The two inequalities are equivalent: replace w with $-w$.

- ▶ Consider the triangle with vertices $0, z$ and $z + w$.
- ▶ The sum of any two sides lengths is \geq the third side length
- ▶ Use this for all three sides and repackage



Worked example of triangle inequality

Show that for z with $|z + 2j| \leq 1$ we have

$$\frac{1}{\sqrt{5} + 1} \leq \left| \frac{z}{z + 1} \right| \leq \frac{3}{\sqrt{5} - 1}$$

General Hints and ideas

- ▶ Treat denominator and numerator separately
- ▶ Rewrite what you want in terms of what you know
- ▶ Be careful of the direction of your inequalities
- ▶ Drawing a picture can help...

Clicker Session
Turning Point app or
ttpoll.eu



Z < W



|Z| < |W|

Section 2: Special functions

We won't sweat this, but defining $\exp(z) = e^z$ properly is tricky...

Definition

$$\exp(x) = 1 + z + z^2/2 + z^3/3! + \dots = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

Binomial theorem means $\exp(z)$ behaves like an exponential:

$$\exp(z + w) = \exp(z) \exp(w)$$

And using Taylor series for $\sin(z)$ and $\cos(z)$ Euler's formula follows, and for $z = x + iy$

$$\exp(z) = e^x (\cos(y) + i \sin(y))$$

So $\exp(z)$ is periodic with period $2\pi i$: $\exp(z + k2\pi i) = \exp(z)$.

Other functions

Using Euler's formula, for real z we have

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \quad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

The right hand sides make sense for complex z , however, and so we can use these as the definition for $\sin(z)$ and $\cos(z)$ for $z \in \mathbb{C}$.

- ▶ Still have $\cos^2(z) + \sin^2(z) = 1$
- ▶ No longer have $|\sin(z)| \leq 1$!

Hyperbolic versions: leave out the i

$$\cosh(z) = \frac{e^z + e^{-z}}{2} \quad \sinh(z) = \frac{e^z - e^{-z}}{2}$$

So $\sin(z)$ and $\cos(z)$ are periodic with period 2π .

Section 2.3 Worked examples

Example 1

Find M such that

$$\left| \frac{e^z + \cos(z)}{z + 6} \right| \leq M$$

for all z with $|z| = 1$.

Example 2

Find the zeroes of $\cos(z)$