## Today's plan:

- 1. Finish Section 3: Line integrals
- 2. Cover Section 4: Notation about sets

#### But first:

Any questions?

## Actually computing line integrals

$$\int_{\gamma} f(z) dz := \int_{a}^{b} f(z(t)) z'(t) dt$$

#### Example

Let  $\gamma$  be the path that's a straight line from 0 to 1, and then a straight line from 1 to 2+i. Compute:

$$\int_{\gamma} \operatorname{Re}(z) dz$$

$$\int_{\gamma} \operatorname{Im}(z) dz$$

$$\int_{\gamma} z dz$$

## The most important example

Recall:  $C_r(a)$  is the anti-clockwise circle of radius r around a.

#### An mysterious computation:

Let  $n \in \mathbb{Z}$ 

$$\int_{C_r(a)} \frac{1}{(z-a)^n} dz = \begin{cases} 0 & n \neq 1 \\ 2\pi i & n = 1 \end{cases}$$

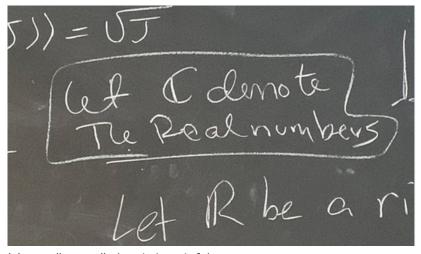
Independent of a and r, works for n negative, too!

#### Coming attractions - conceptual explanation!

- Antiderivatives explain why the answer is zero unless n=1
- Cauchy's theorem explains why it's independent of r
- Residue theorem reduces any integral to this computation!

# Clicker Session Turning Point app or ttpoll.eu

#### Seen on twitter - "Chaotic evil" maths



It's not "wrong", but it is painful.

Some definitions in the notes are similar...

## Types of Sets – a topology taster

The following Definition in the notes is NOT STANDARD USAGE. I won't use it.

#### Definition (4.1 in notes)

A *neighbourhood* of  $z_0 \in \mathbb{C}$  is an open disc about  $z_0$ , i.e., it's of the form

$$\{z\in\mathbb{C}:|z-z_0|<\delta\}$$

for some  $\delta > 0$ .

## Definition (4.2 in notes. This definition is standard)

A set  $D \subseteq \mathbb{C}$  is said to be *open* if for each point  $z_0 \in D$  there's an open disc contained in D and containing  $z_0$ .

Easier to think in terms of pictures...

"Can't be cut in two" vs. "Can get from place to place"

Two possible intuitions behind being connected.

#### Definition (Connected)

A subset  $X \subseteq \mathbb{C}$  is *connected* if we can't find two nonempty open sets  $U, V \subset \mathbb{C}$  with  $U \cap V = \emptyset$  and  $X \subset U \cup V$ .

Intuition: If  $X \subset U \cap V$ , then  $X \cap U, X \cap V$  cuts X into two pieces.

#### Definition (Path-connected)

A subset  $X\subseteq\mathbb{C}$  is called *path-connected* if for any two points  $x,y\in X$  we can find a path  $\gamma\in X$  with initial point x and final point y

- ▶ In general, path-connected ⇒ connected, but not converse
- ▶ If X is open, X is connected  $\iff X$  is path-connected.

#### A little bit more

Many of our Theorems are going to hold for nice open sets, so we develop shorthand for recording this.

#### Definition (4.4 in notes)

A non-empty, open, connected set is called a region.

## Definition (4.5 in notes)

A region D is said to be *simply connected* if it has no 'holes'; i.e., if every point in the interior of any simple contour in D is contained in D.

#### Mathematical culture:

We're implicitly using the Jordan Curve theorem – that a simple curve closed in the plane has an inside and an outside.

#### Why is this hard?

# Pick a point in the middle – is it inside, or outside?

